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Class: -X | **Topic: - Polynomial** | **Subject: -Mathematics**

Some important formulas and Rules

Important Algebraic Formulas

1. $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

2. $\alpha^2 - \beta^2 = (\alpha+\beta)(\alpha-\beta)$

3. $(\alpha-\beta) = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

4. $(\alpha^3+\beta^3) = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

5. $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

6. $\alpha^4 + \beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$

7. $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$

Some important properties related to zeroes

$P(x) = ax^2+bx+c$

1. If sign of product of ac is positive and sign of b is positive then both zeroes are negative

$P(x) = x^2 + 5x + 6 = x^2 + 3x + 2x + 6$
$= x(x+3) + 2(x+3) = (x+3)(x+2)$
$P(x)=0;\ (x+3)=0$ or $(x+2)=0 \therefore x - 3$ or -2

2. If sign of product of ac is positive and sign of b is negative then both zeroes are positive

$P(x) = x^2 - 5x + 6 = x^2 - 3x - 2x + 6$
$= x(x-3) - 2(x-3) = (x-3)(x-2)$
$P(x)=0;\ (x-3)=0$ or $(x-2)=0 \therefore x = 3$ or 2

3. If sign of product of ac is negative and sign of b is negative or positive then both zeroes are opposite sign.

$P(x) = x^2 - x - 6 = x^2 - 3x + 2x - 6$
$= x(x-3) + 2(x-3) = (x-3)(x+2)$
$P(x)=0;\ (x-3)=0$ or $(x+2)=0 \therefore x = 3$ or -2

Do Your Self

1) If α and β are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$ then evaluate:

a) $\alpha^2 + \beta^2$

b) $\alpha^3 + \beta^3$

c) $\alpha^4 + \beta^4$

d) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

e) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

f) $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$

g) $\alpha - \beta$

2) Find the zeros of each of the following quadratic polynomial and verify the relationship between the zeros and their coefficients:

a) $f(x) = x^2 - 2x - 8$

b) $p(x)=6x^2 - 3 - 7x$

c) $p(x) = x^2 + 2\sqrt{2}x - 6$

d) $q(x) = \sqrt{3}x^2 + 10x + 7\sqrt{3}$

e) $g(x) = a(x^2 + 1) - x(a^2 + 1)$

f) $h(t) = t^2 - 15$

g) $p(y) = 5y^2 - 7y + 1$

Example: - If α *and* β are the zeros of the quadratic polynomial $p(x) = ax^2 + bx + c$ then evaluate $\alpha^2 + \beta^2$

$$p(x) = ax^2 + bx + c$$

$$\alpha + \beta = \frac{-b}{a}, \quad \alpha\beta = \frac{c}{a}$$

$$\text{We know that } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{-b}{a}\right)^2 - 2\frac{c}{a} = \frac{b^2 - 2ca}{a^2}$$ Answer

Example: - Find the zeros of each of the following quadratic polynomial and verify the relationship between the zeros and their coefficients:

$f(x) = x^2 - 2x - 8$

$f(x) = x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= x(x - 4) + 2(x - 4)$

$= (x - 4)(x + 2)$

Now $f(0) = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow (x - 4) = 0$ or $(x + 2) = 0$

$\therefore x = 4$ or -2 Answer

Relationship between the zeros and their coefficients:-

Sum of zeros = $\frac{-b}{a}$

$4 + (-2) = \frac{-(-2)}{1}$

$\therefore 2 = 2$ Verified

Product of zeroes = $\frac{c}{a}$

$4 \times (-2) = \frac{-8}{1}$

$\therefore -8 = -8$ Verified